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$\mathbb{Z}^2$  is a free  $\mathbb{Z}$ -module of rank 2. Let  $\mathcal{L}$  be a lattice in  $\mathbb{Z}^2$ . The quotient  $\mathbb{Z}^2 / \mathcal{L}$  is a finite abelian group. The order of  $\mathbb{Z}^2 / \mathcal{L}$  is the area of the fundamental parallelogram of  $\mathcal{L}$ . If  $\mathcal{L}$  is generated by  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ , then the order is  $|\det \begin{pmatrix} a & b \\ c & d \end{pmatrix}|$ . For example, if  $\mathcal{L} = 2\mathbb{Z} \times 2\mathbb{Z}$ , then the order is  $2 \times 2 = 4$ .

The index of  $\mathcal{L}$  in  $\mathbb{Z}^2$  is the number of cosets of  $\mathcal{L}$  in  $\mathbb{Z}^2$ . This is equal to the order of  $\mathbb{Z}^2 / \mathcal{L}$ .

1. Let  $\mathcal{L}$  be a lattice in  $\mathbb{Z}^2$ . The index of  $\mathcal{L}$  in  $\mathbb{Z}^2$  is  $|\det \begin{pmatrix} a & b \\ c & d \end{pmatrix}|$ , where  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$  is a matrix whose columns are the generators of  $\mathcal{L}$ .

2. ...

3. ...

4. Let  $\mathcal{L}$  be a lattice in  $\mathbb{Z}^2$ . The index of  $\mathcal{L}$  in  $\mathbb{Z}^2$  is  $|\det \begin{pmatrix} a & b \\ c & d \end{pmatrix}|$ , where  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$  is a matrix whose columns are the generators of  $\mathcal{L}$ .

5. Let  $\mathcal{L}$  be a lattice in  $\mathbb{Z}^2$ . The index of  $\mathcal{L}$  in  $\mathbb{Z}^2$  is  $|\det \begin{pmatrix} a & b \\ c & d \end{pmatrix}|$ , where  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$  is a matrix whose columns are the generators of  $\mathcal{L}$ .

Let  $\mathcal{L}$  be a lattice in  $\mathbb{Z}^2$ . The index of  $\mathcal{L}$  in  $\mathbb{Z}^2$  is  $|\det \begin{pmatrix} a & b \\ c & d \end{pmatrix}|$ , where  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$  is a matrix whose columns are the generators of  $\mathcal{L}$ .